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# Investigation of the unsteady-state heat transfer in the region of a stagnation point in plane and axisymmetric flows

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**Abstract**—The paper presents the results of experimental and theoretical investigation of heat transfer on the upstream side of a sphere and transversely-oriented cylinder immersed in a gas flow under the unsteady-state conditions resulting from a stepwise change in the flow (or body) temperature.

## INTRODUCTION

A SURVEY of unsteady-state heat transfer experimental studies, including those carried out by the present authors, has shown [1] that a number of works report the dependence of the heat transfer coefficient in unsteady-state conditions on time, body dimensions in the direction of heat propagation and thermophysical properties of the body material which exchanges heat with a liquid or gas flow. The results of experimental studies can be explained qualitatively using the process model based on thermal boundary layer formation [1]. Not all of the conclusions of these studies agree well amongst themselves. This applies to the duration of the unsteady-state process, to the relationship between the heat transfer coefficient in unsteady- and steady-state conditions and to the dependence of the unsteady-state heat transfer coefficient on the properties of the body [1, 2].

The controversy in the results obtained has stimulated further investigations of heat transfer in unsteady-state conditions.

In one study [3] the heat transfer coefficient around the perimeter of an air-cooled hollow copper cylinder in a crossflow was investigated. The results obtained can be presented by the expressions

$$\frac{Nu_{un}}{Nu_{st}} = 1.0 + \frac{600-382\varphi}{\tau} \tag{1}$$

for $\varphi = 0-\pi/2$ and

$$\frac{Nu_{un}}{Nu_{st}} = 1.0 + \frac{-500+319\varphi}{\tau} \tag{2}$$

for $\varphi = \pi/2-\pi$, where $\tau = \tau^* \cdot U_\infty/R$.

The developed Nusselt numbers $Nu_{st}$ for the upstream side of the cylinder, to which $Nu_{un}$ tends to approach with time, agree within 7% with Kruzhilin's relation [4]:

$$Nu_{st} = 1.04\, Re^{0.5} Pr^{0.33}. \tag{3}$$

✠ Deceased.

The flow velocity varied from 5 to 20 $ms^{-1}$, the air temperature was 20°C and the initial temperature of the cylinder was 80°C. Heat transfer around the perimeter was prevented by the cylinder sectors having been thermally insulated. The confidence interval for the measurement of the heat transfer coefficient was $\pm 8\%$; the confidence probability was 0.85. The error introduced by the approximation of the results of measurement by formulae (1) and (2) was less than 4%.

Measurements of the heat flux from a plasma jet to an axisymmetric spherically blunted body in the stagnation point region, when this body is inserted into the jet in a number of ways including the case where a shutter is used to introduce a probe into the flow, have shown [5] that the heat flux may vary during the first hundredths–tenths of a second. Moreover, this phenomenon cannot be explained by a change of the parameters over the jet radius or by the process of shutter removal. For the measurement of the heat flux, a probe has been suggested which represents a model of a semi-infinite body in which the temperature is measured near the working surface. The heat flux on the surface was calculated from the temperature measured, following the procedure given in [6]. The checking of the method of heat flux determination with the aid of calculation models and experiments has proved its validity for the conditions considered. The confidence interval for the measurement of the heat flux with the use of this method is within $\pm 12\%$, with the confidence probability being 0.85.

A combined probe allowing simultaneous measurement of the time-variable heat flux and the stagnation pressure has been suggested [7]. It transpired that the observed change in heat flux is mainly associated with variation of the gas flow parameters and of stagnation pressure, in particular (Fig. 1) at constant pressure in the discharge chamber of an electric arc heater.

In order to assess the role of the processes of formation and rearrangement of the thermal boundary layer and also of the properties of a body in the unsteady-state heat transfer, the problems of some limiting cases of the unsteady-state heat transfer will be

## NOMENCLATURE

| | |
|---|---|
| $c\rho$ | volumetric heat capacity |
| $D$ | diameter of sphere or cylinder |
| $f$ | function determined by expression $\Psi = \Psi(\beta\nu)^{1/2}xf(\eta)$ |
| $k$ | part of the period during which heat transfer occurs |
| $K_\lambda$ | $\lambda_b/\lambda_f$ |
| $K_{c\rho}$ | $(c\rho)_b/(c\rho)_f$ |
| $Nu_{un}$, $Nu_{st}$ | Nusselt numbers in unsteady-state and steady-state conditions |
| $Nu_{q\,st}$ | developed Nusselt number at the end of unsteady-state process |
| $q$ | specific heat flux at the surface |
| $R$ | radius of sphere or cylinder |
| $t_e - t_0$ | temperature jump outside the boundary layer at time zero |
| $t_0$ | initial temperature, equations (5)–(7); temperature far from the surface, equations (25)–(29) |
| $t_e$ | temperature at the outer edge of the boundary layer, equations (5)–(7); temperature of body surface at initial time of unsteady-state process, equations (25)–(29) |
| $u$ | velocity component along axis $x$ |
| $U_\infty$ | free-stream velocity |
| $x, y$ | longitudinal and transverse coordinate, respectively. |

Greek symbols

| | |
|---|---|
| $\beta$ | velocity gradient at stagnation point related to velocity $u_e$ as $u_e = \beta \cdot x$ |
| $\delta$ | wall thickness |
| $\theta$ | $= (t-t_0)/(t_e-t_0)$ |
| $\eta$ | $= y(\beta/\nu)^{1/2}$ in equations (4)–(8) |
| $\eta$ | $= y(2^i\rho/\nu)^{1/2}$ in equations (24)–(29) |
| $\tau^*$ | dimensional time |
| $\tau$ | $= \beta \cdot \tau^* Pr$ in equation (4)–(8) |
| $\tau$ | $= 2^i\beta\tau^*/Pr$ in equations (24)–(27) |
| $\Psi$ | stream function. |

Subscripts

| | |
|---|---|
| b | body |
| bound | boundary |
| e | edge of the boundary layer |
| f | fluid |
| $\infty$ | far from the surface. |

considered below. In addition the consequences of the unsteady-state nature of the process, which are of certain practical importance, will be analyzed.

The problem of the unsteady-state heat transfer in the region of the stagnation point of a blunt body has been considered [8–10]. The unsteady-state process originating after a stepwise change in the body surface temperature, heat flux on the surface, stepwise change in the temperature at a certain distance from the surface at the zero surface temperature, stepwise change in the flow temperature for the case of constant surface temperature and zero heat flux at the downstream surface of a thin plate at the constant temperature through the plate thickness at each time instant were investigated. In all of the cases the problem was solved in non-conjugate formulation, which made it impossible to reveal the role of the body in the heat transfer process except for the last case where heat transfer in the body was allowed for approximately. The investigations [8–10] showed that the heat transfer coefficient in the unsteady-state conditions considered is the function of time. The relaxation time of the heat transfer characteristics depends on the number $Pr$ and flow velocity.

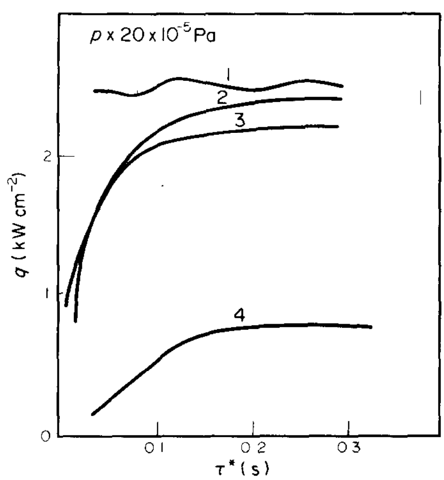


FIG. 1. The time dependence of the parameters in a jet and discharge chamber: 1, excess pressure in the chamber; 2, 3, heat flux to a barrier; 2, probe with an opening; 3, probe without an opening; 4, stagnation pressure in a jet.

In order to explain an experimentally observed [11] increase in the heat flux from zero up to the developed value on the introduction of a probe into a high-temperature gas flow, the following scheme of the process can be suggested. At the initial time the gas temperature varies in a jumpwise fashion throughout the entire flow except for a certain region near the surface; then the cold layer heats up right through causing the observed change in the heat flux. To check this assumption, a conjugate unsteady-state heat transfer problem for the stagnation point region of an axisymmetric spherically blunted body will be considered.

The problem is solved under the general assumptions made in previous studies [8–10]. These are: the body is streamlined by a subsonic steady-state laminar incompressible liquid flow with constant properties; energy dissipation in the flow is negligibly small. The energy equation term, which represents the convective transport of heat along the surface, is ignored for the reason that the spatial derivative of temperature oriented along the surface is equal to zero at the stagnation point.

The heat transfer in the body is assumed to be one-dimensional because the probes used to investigate the unsteady-state heat transfer in the stagnation point region are usually metallic rods with a heat-insulated cylindrical surface to provide one-dimensional heat propagation along the axis. This is satisfactorily contrived in practice [12, 13].

With the above assumptions taken into account, the problem setting in dimensionless form includes the following equations:

*momentum*

$$f'^2 - 2ff'' = 1 + f''', \tag{4}$$

*heat conduction in a boundary layer*

$$\frac{\partial \theta}{\partial \tau} = 2\,Pr\,f\,\frac{\partial \theta}{\partial \eta} + \frac{\partial^2 \theta}{\partial \eta^2}, \tag{5}$$

*heat conduction in a body*

$$\frac{\partial \theta}{\partial \tau} = \frac{a_b}{a_f}\left(\frac{\partial^2 \theta}{\partial \eta^2}\right), \tag{6}$$

*boundary-value conditions*

$$\theta(\eta, 0) = 0,\quad \theta(\eta_e, \tau) = 1,\quad \theta(\eta_{-\infty}, \tau) = 0. \tag{7}$$

$$\theta_f(0,\tau) = \theta_b(0,\tau),\quad \frac{\partial \theta_f(0,\tau)}{\partial \eta} = \frac{\lambda_b}{\lambda_f}\cdot\frac{\partial \theta_b(0,\tau)}{\partial \eta}$$
$$f(0) = f'(0) = 0,\quad f'(\eta_e) = 1. \tag{8}$$

The solution of equation (4) satisfying conditions (8) is known [14] and can be approximated within 5% by the relations

$$f = 0.469\,\eta^{1.8},\quad 0 \leqslant \eta \leqslant 1.4 \quad f = \eta - 0.53,\quad \eta > 1.4. \tag{9}$$

The solution of the system of equations (5)–(7) determines the temperature fields from which the unsteady-state heat transfer coefficient and the number $Nu$ can be calculated. Bearing in mind that for a sphere $Re = \beta D^2/3\nu$, one obtains

$$Nu = \frac{\theta_f(0,\tau)\sqrt{3}\,\sqrt{Re}}{1-\theta_f(0,\tau)}. \tag{10}$$

It follows from equations (5)–(7) and (10) that

$$Nu_{un}/\sqrt{Re} = \varphi(Pr, K_\lambda, K_{cp}, U_\infty, \tau^*/D), \tag{11}$$

i.e. the unsteady-state heat transfer coefficient depends, besides $Pr$ and $Re$, on the ratio between the thermophysical coefficients of the body and liquid and on the dimensionless time.

The omission of the time-involving terms from equations (4)–(8) yields the statement of the steady-state conjugate problem which corresponds to the above unsteady-state one. That the steady-state problem could be solved numerically with the use of a computer, the method of opposite factorizations [15] has been employed. The complex $Nu_{st}/\sqrt{Re}$ is determined by the parameters $K_\lambda$ and $Pr$, *i.e.* just as in the unsteady-state case it is the function of the liquid and body properties. In the steady-state case the complex $Nu/\sqrt{Re}$ is independent of $K_{cp}$.

The calculations performed for $K_\lambda$ varying from 1 to $10^4$ and for $Pr$ within the range from 0.01 to 100 have shown that a number of parameter ranges can be isolated with its own characteristic trend of $Nu/\sqrt{Re}$ variation with $Pr$ and $K_\lambda$. In the first region ($K_\lambda \cong$ 10–$10^3$, $Pr \leqslant 10^{-1}$), $Nu_{st}/\sqrt{Re}$ is independent of $Pr$. With an increase of $K_\lambda$ above 500–1000, $Nu_{st}/\sqrt{Re}$ practically ceases to depend on this parameter also. In this region $Nu_{st}/\sqrt{Re}$ is determined by the expression

$$Nu_{s}/\sqrt{Re} = 1.25[1 - \exp(-0.772 - 0.0255\,K_\lambda)]. \tag{12}$$

The second region comprises the change of $K_\lambda$ within 10–$10^3$ and of $Pr$ within $10^{-1}$–$10^2$. The dependence of $Nu_{st}/\sqrt{Re}$ on $K_\lambda$ and $Pr$ in this region is satisfactorily described by the expressions of the form

$$\frac{Nu_{st}}{\sqrt{Re}} = a[\log(10^2\,Pr)]^b + C, \tag{13}$$

where the quantity $C$ is equal to the corresponding value of $Nu_{st}/\sqrt{Re}$ in the first region, equation (12). The coefficients $a$ and $b$ for $K_\lambda \geqslant 20$ are equal to

$$a = 0.151 + 0.1\{1 - \exp[-2.25(\log K_\lambda - 1.3)]\},$$
$$b = 2.0 + 0.72\{1 - \exp[-1.93(\log K_\lambda - 1.3)]\}. \tag{14}$$

At $K_\lambda$ above 500–1000, $Nu_{st}/\sqrt{Re}$ does not depend on $K_\lambda$ in this region too. More precisely, the boundary separating the region, in which $Nu_{st}/\sqrt{Re}$ does not depend on $K_\lambda$ within the region of $Pr$ = 0.01–100, is determined by the expression

$$(Nu_{st}/\sqrt{Re})_{bound} = 5.2\log K_\lambda - 12. \tag{15}$$

The change of $Nu/\sqrt{Re}$ in the third region ($K_\lambda$ = 1–10, $Pr$ = 0.01–100) is approximated by the expressions for $Pr \cong$ 0.01–0.5

$$Nu_{st}/\sqrt{Re} = 1.62^{\log K_\lambda} - 0.85; \tag{16}$$

for $Pr \cong 0.5$–$100$

$$Nu_{st}/\sqrt{Re} = (1.71\,Pr^{0.069})^{\log K_\lambda} - 0.85. \quad (17)$$

The error of approximation by formula (12) is 6.3%; by formula (13), 8%; by formulae (16) and (17), 3%.

The steady-state problem of heat transfer in the stagnation point region was solved by Sibulkin [16]. The assumption of zero surface temperature, adopted in work [16], approaches most closely the case of large values of $K_\lambda$ in the present work. The value $Nu/\sqrt{Re} = 1.32$, obtained [16] for $Pr = 1$, agrees satisfactorily with the value found from the solution of the conjugate problem at $K_\lambda > 100$.

The results of computer calculations of the unsteady-state heat transfer coefficients for $Pr = 0.7$, $K_\lambda = (1$–$1.5)10^4$, $K_{cp} = 2000$ and 6000 cover the air flow interaction at low and high temperatures (if mean values of the parameters are considered) with the obstacles made of metals and some heat insulators. For air and nitrogen plasmas at $T = 4000$–$9000$ K in combination with metal obstacles, $K_\lambda = 10$–$1000$.

As is to be expected for the process scheme considered, $Nu/\sqrt{Re}$ increases in the initial period approaching a constant value over the time interval studied (Fig. 2). The exceptions are the values of $K_\lambda < 10$ at which, after the attainment of the maximum, the dependence becomes a decreasing one. The maximum value of $Nu/\sqrt{Re}$ depends on the parameter $K_\lambda$ up to its values of order $10^3$, after which the function $Nu/\sqrt{Re} = f(\tau)$ does not practically change with an increase of $K_\lambda$. The calculated relations are only slightly influenced by changes in the parameter $K_{cp}$ within a practically encountered region at $Pr = 0.7$ (Fig. 2). The initial growth of $Nu/\sqrt{Re}$ in time is due to the temperature profile rearrangement in the boundary as a result of its heating. The increase of $Nu/\sqrt{Re}$ with $K_\lambda$ is due to a more intense heat dissipation into the interior of the body with an increase in its relative thermal conductivity. In this case, the increase of the heat transfer rate as a result of the higher thermal conductivity of the body reaches the limit at $K_\lambda \approx 1000$, when the process of heat transfer in the boundary layer becomes a limiting one.

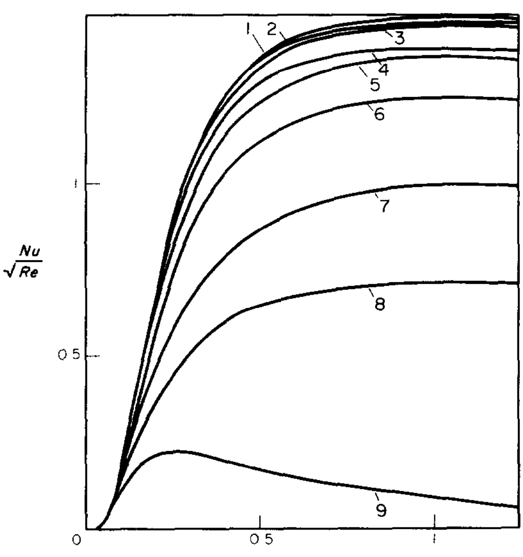


FIG. 2. The complex $Nu/\sqrt{Re}$ vs the dimensionless time $\tau$: $Pr = 0.7$; 1, $K_\lambda = 10^4$–$1.5 \times 10^4$; 2, $10^3$; 3, 500; 4, $10^3$; 5, 100; 6, 50; 7, 20; 8, 10; 9, 1; 1–3, 5–9, $K_{cp} = 2 \times 10^3$–$6 \times 10^3$; 4, 0.9.

The time required for $Nu/\sqrt{Re}$ to attain its maximum value increases with $K_\lambda$ and does not practically depend on $K_{cp}$. The dependence of $\tau_{max}$ on $K_\lambda$ within the range of $K_\lambda$ from 1 to 20000 and of $K_{cp}$ from 2000 to 6000 is described, with the approximation error within 1–10%, by the expression

$$\tau_{max} = 1.4\{1 - \exp[-(0.9 \log K_\lambda + 0.2)]\}. \quad (18)$$

The error 10% relates to the value of $Nu/\sqrt{Re}$ at $K_\lambda = 1$.

The dependence of $Nu/\sqrt{Re}$ on time up to the instant corresponding to the maximum value of this function, can be approximated at $\tau \gtrsim 0.1$ and $Pr = 0.7$ by the expression

$$Nu/\sqrt{Re} = (Nu/\sqrt{Re})_{max}[1 - \exp(-b\tau + 0.6)], \quad (19)$$

where

$$(Nu/\sqrt{Re})_{max} = 1.5[1 - \exp(1.5 - 2 \log K_\lambda)] \text{ at } K_\lambda \geqslant 20,$$

$$(Nu/\sqrt{Re})_{max} = 0.22(3.32)^{\log K_\lambda} \text{ at } 1 \leqslant K_\lambda < 20,$$

$$b = 6.17[1 - \exp(1.8 - 2.53 \log K_\lambda)] \text{ at } K_\lambda \geqslant 20.$$

The error of approximation by formula (19) is from 10% at small values of time and $K_\lambda$ to less than 1% at the end of the process.

In order to estimate the time for which the unsteady-state heat transfer coefficient attains the maximum value at $Pr = 2$–$7$ (Fig. 3), $K_{cp} = 0.3$–$0.9$; $K_\lambda = 1$–$10^3$, the following formula can be used

$$\tau_{max} = (0.26 + 0.07\,K_{cp}) \times [\exp(-0.16\,Pr)] \log K_\lambda + 0.185, \quad (20)$$

the error of approximation by which is 2% at $Pr = 7$ and 10% at $Pr = 2$.

The maximum value of the complex $(Nu_{un}/\sqrt{Re})_{max}$ is determined, with the error of 5%, by the expression

$$(Nu_{un}/\sqrt{Re})_{max} = (0.15\,Pr + 1.35) \times \{1 - \exp[-(0.0121 + 0.00517\,K_{cp} - 0.00033\,Pr) \times K_\lambda + 0.36 - 0.01\,Pr]\}. \quad (21)$$

The introduction of the relative values of $Nu_{un}/(Nu_{un})_{max}$ and $\bar{\tau} = \tau/(\tau_{max} - 0.05/\tau_{max})$ has made it possible to comprise the calculated dependencies into a single one, which at $\tau \cong 0.08$–$1$ is satisfactorily

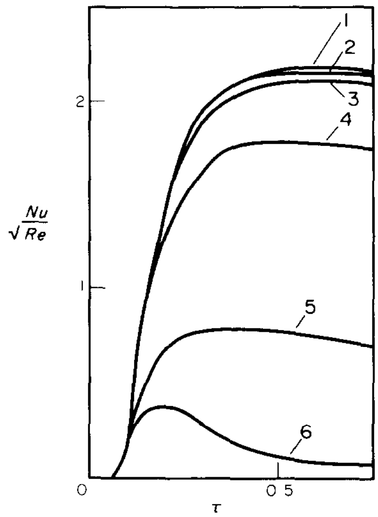


FIG. 3. The complex $Nu/\sqrt{Re}$ vs the dimensionless time $\tau$: $Pr = 5$; $K_{c\rho} = 0.9$; 1, $K_\lambda = 1000$; 2, 800; 3, 500; 4, 100; 5, 10; 6, 1.

represented by the expression

$$Nu_{un}/(Nu_{un})_{max} = 1 - \exp(0.35 - 5.46\,\tau). \quad (22)$$

Formulae (20)–(22) describe the unsteady-state heat transfer coefficient for water flow interaction with blunt bodies made of metals and insulators in the stagnation point region when the flow temperature outside the boundary layer changes in a stepwise fashion from that initially equal to the body temperature.

The comparison of the conjugate problem solutions obtained for the steady-state and unsteady-state cases has shown that in the initial period of the unsteady-state heat transfer process at the constant values of the unperturbed flow parameters, equal to the parameters of the steady-state case, the heat transfer coefficient may differ by several times from its steady-state value. After the boundary layer has been heated, these values become sufficiently consistent.

In the notation adopted in the present work, the dimensional time is $\tau^* = Pr\,R\tau/3U_\infty$. If it is assumed that $R = 10^{-2}$ m, which corresponds to the dimensions of the bodies used in the experiments, then for air at $U_\infty = 10^3$ ms$^{-1}$ the time for the heat transfer coefficient to attain its maximum value, which has the order of the time for layer heating, is about $10^{-6}$ s. For water at $U_\infty = 1$ ms$^{-1}$, this time is equal to $10^{-2}$ s.

Above, the limiting case of the process has been considered which consists in the heating-up of the liquid layer which directly adjoins the body and which initially has the temperature equal to that of the body.

An alternative limiting case is also possible. When a body is introduced into the flow with the temperature different from that of the body, the following scheme of the unsteady-state heat transfer process is feasible. If the heating (cooling) of the adjacent thin liquid (gas) layer is ignored, then it is possible to regard that at the initial time instant there has occurred a temperature jump throughout the entire liquid volume up to the body surface, and then equalization of the temperatures within the body and the flow takes place. The problem consists in determining the heat transfer characteristics at the stagnation point under these unsteady-state conditions. As before, a subsonic laminar steady flow is considered, the liquid properties are assumed to be constant. It is also assumed that the body at the stagnation point is a hollow sphere made of heat conducting material of thickness $\delta$, so that the temperature drop across its thickness can be ignored. This is contrived experimentally. The above condition allows the wall heat conduction equation to be replaced by the expression

$$q = \delta c\rho\, dt/d\tau^*, \quad (23)$$

which takes into account the time change of the wall temperature. The heat flux on the inner wall surface is taken to be zero. This condition is met in heat flux probes [17, 18].

In view of the above, the statement of the problem in dimensionless form includes the *momentum equation*

$$f''' + ff'' + \frac{1}{2^i}(1 - f'^2) = 0 \quad (24)$$

(where $i = 0$ for the frontal line region of a cylinder in a cross flow, $i = 1$ for the frontal point of a sphere), the solution of which subject to the boundary conditions $f(0) = f'(0) = 0$ and $f'(\infty) = 1$ is known [8];

*the boundary layer heat conduction equation*

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial\eta^2} + Pr f\frac{\partial\theta}{\partial\eta}; \quad (25)$$

*the boundary conditions*

$$\tau < \tau_1, \quad \theta = 0, \quad \tau = \tau_1, \quad \theta_b = 1, \quad (26)$$

$$\tau > \tau_1, \quad \theta_{f,\eta=0} = \theta_{b,\eta=0}, \quad (27)$$

$$-\left(\frac{\partial\theta_f}{\partial\eta}\right)_{\eta=0} = P\,Re^{1/2}\frac{\delta}{D}K_{c\rho}\frac{d\theta_b}{d\tau}, \quad (28)$$

$$\theta_{f,\eta=\infty} = 0,$$

$P = 2$ for a cylinder, $P = \sqrt{6}$ for a sphere.

The quantity sought is $Nu$ defined as

$$\frac{Nu}{\sqrt{Re}} = -\frac{P}{\theta_b}\left(\frac{\partial\theta_f}{\partial\eta}\right)_{\eta=0}. \quad (29)$$

It follows from equations (24)–(29) that

$$\frac{Nu}{\sqrt{Re}} = \varphi(Pr, Re, \delta/D, (c\rho)_b/(c\rho)_f, \beta\tau^*), \quad (30)$$

i.e. in addition to such ordinary parameters as $Pr$ and $Re$, the number $Nu$ in the model considered depends also on the dimensionless wall thickness, the ratio between the products of the specific heat by the body material and liquid density and on dimensionless time.

The calculations were made on a computer for: $Pr = 0.015$–$50$; $Re = 10^2$–$10^6$; $\delta/D = 0.001$–$0.2$;

$K_{cp} = 0.3$–$0.9$ (for $Pr > 2$) and $K_{cp} = 2\times10^3$–$6\times10^3$ (for $Pr \leqslant 0.7$). The values of the parameter $K_{cp}$ correspond to combinations of bodies made from metals and some insulators with the flows of gases, water and some more viscous liquids.

The calculations have shown that the complex $Nu/\sqrt{Re}$ depends substantially on time, $Pr$ number and the ratio $\delta/D$ and, to a lesser extent, on $Re$ and $K_{cp}$ (Fig. 4). The dependence of $Nu$ on $Re$, equal for the steady-state heat transfer in laminar flow to $Nu \sim Re^{0.5}$, will be different in unsteady-state conditions. As time progresses, $Nu \sim Re^{0.5}$ decreases approaching a constant (quasi-steady) value. The increase of $Pr$ and $\delta/D$ leads to an increase of $Nu/\sqrt{Re}$. The quasi-steady value of $Nu_{q\,st}/\sqrt{Re}$ increases with $\delta/D$, $Re$, approaching a constant value.

The approximation of the limiting dependence of $Nu/\sqrt{Re}$ on $Pr$ for $\delta/D \geqslant 0.2$, $Re \geqslant 10^5$, $K_{cp} = 0.3$–$0.9$ leads to the following expression for the frontal point of the sphere

$$Nu_{q\,st}/\sqrt{Re} = 1.38\,Pr^{0.35}, \tag{31}$$

which agrees satisfactorily with the dependence for the case of steady-state heat transfer [16]:

$$Nu_{st}/\sqrt{Re} = 1.32\,Pr^{0.4}. \tag{32}$$

Formulae (31) and (32) approximate the results of numerical calculation for $Pr = 0.7$–$50$ and $0.7$–$2.0$, respectively.

The dependence of $Nu_{q\,st}/\sqrt{Re}$ on $Pr$ for the frontal line of a cylinder at $\delta/D \geqslant 0.1$ and $Re \geqslant 10^4$ is

$$\frac{Nu_{q\,st}}{\sqrt{Re}} = 1.1\,Pr^{0.37} \quad Pr = 0.7 \div 10. \tag{33}$$

Expression (33) agrees satisfactorily with formulae

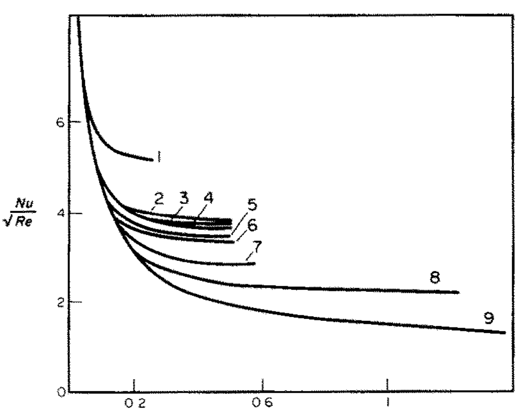


FIG. 4. Examples of the dependence of the complex $Nu/\sqrt{Re}$ on the dimensionless time $\tau$: 1, 4, 7, 8, 9, $Re = 10^3$; $\delta/D = 0.2$; $K_{cp} = 0.3$; 1, $Pr = 50$; 4, 20; 7, 10; 8, 5; 9, 0.7; 2, 3, 5, $Pr = 20$; $\delta/D = 0.2$; 2, $Re = 10^6$; $K_{cp} = 0.3$; 3, $Re = 10^3$; $K_{cp} = 0.9$; 5, $Re = 10^2$, $K_{cp} = 0.3$; 6, $Pr = 20$, $K_{cp} = 0.3$; $Re = 10^3$, $\delta/D = 0.05$.

(34) and (35) for the steady-state heat transfer case [4, 19]

$$\frac{Nu_{st}}{\sqrt{Re}} = 1.04\,Pr^{0.33} \tag{34}$$

$$\frac{Nu_{st}}{\sqrt{Re}} = 1.14\,Pr^{0.35}. \tag{35}$$

A special feature of the heat transfer in unsteady-state conditions for a sphere at $Pr \leqslant 1$ is that at $Re = 10^2$–$10^6$; $K_{cp} = 0.3$–$6\times10^3$; $\delta/D = 0.01$–$0.3$; $Pr = 0.015$–$1$ the above parameters do not practically influence the dependence of $Nu/\sqrt{Re}$ on the dimensionless time. For the values of $\tau$ form 0.025 to 1.4, when the variation of $Nu/\sqrt{Re}$ in time practically ceases, this dependence is approximated with a 5% error by the expression

$$\frac{Nu}{\sqrt{Re}} = 1.34[1-\exp(-0.15-1.96\tau)]^{-1}, \tag{36}$$

when $\tau < 0.025$, $Nu/\sqrt{Re} \cong 0.18/\tau$.

In order to determine the values of the complex $Nu/\sqrt{Re}$ within the ranges of $Pr$ from 2 to 50, $\delta/D$ from 0.01 to 0.2, $Re$ from $10^2$ to $10^6$, and of $K_{cp}$ from 0.3 to 0.9, the following formulae can be used

$$\frac{Nu}{\sqrt{Re}} = \frac{Nu_{q\,st}}{\sqrt{Re}} \times \{1-\exp[1.8(1-e^{120\delta/D})\,Pr^{0.72}\tau]\}^{-1}, \tag{37}$$

$$\frac{Nu_{qst}}{\sqrt{Re}} = 1.44\,Pr^{0.35}\{1-\exp[-52.5\,\delta/D + 52.5\,\delta/D\,\exp(-0.157\,Pr+0.25)]\} \times(1-1.59\,Re^{-0.54}). \tag{38}$$

For a cylinder with $Pr < 1$, the complex $Nu/\sqrt{Re}$ does not practically depend on $Re$, $\delta/D$, $K_{cp}$ in the range of the parameters studied, and at $Pr = 0.7$, it is described with a less than 10% error, depending on the dimensionless time, by the expression

$$\frac{Nu/\sqrt{Re}}{Nu_{st}/\sqrt{Re}} = \{1-\exp[-(0.12+1.6\tau)]\}^{-1}, \tag{39}$$

where

$$Nu_{st}/\sqrt{Re} = 0.95;\quad Pr = 0.7;\quad Re = 10^2\text{–}10^6;$$
$$\delta/D = 0.01\text{–}0.4;\quad K_{cp} = (3 \div 6)10^3;\quad \tau \geqslant 0.05.$$

In order to determine the unsteady-state heat transfer coefficient in the ranges: $Pr = 2$–$10$; $Re = 10$–$10^7$; $\delta/D = 0.001$–$0.4$; $K_{cp} = 0.3$–$0.9$; $\tau = 0.01$–$3$, the following expression can be used, the error of approximation by which does not exceed 10%:

$$\frac{Nu/\sqrt{Re}}{Nu_{q\,st}/\sqrt{Re}} = \{1-\exp[-0.1 -0.025\,Pr-2(1-e^{-65\delta/D})\tau-0.56\,Pr\tau\}^{-1}, \tag{40}$$

where $Nu_{q\,st}/\sqrt{Re}$ is equal to

$$\frac{Nu_{q\,st}}{\sqrt{Re}} = 1.1\,Pr^{0.37}(1-e^{-\log Re}) \times [1-\exp(-0.1\,Pr+56\,\delta/D)].$$

With increasing $Re$ and $\delta/D$, this expression for $Nu_{q\,st}/\sqrt{Re}$ reduces to equation (33).

A characteristic feature of the quasi-steady solution is that the complex $Nu/\sqrt{Re}$ in the conjugate statement of the problem depends on $Re$ and $\delta/D$ starting from their certain decreasing values.

When $\delta/D < 0.05$ and $Re < 10^3$, the dependence of $Nu_{st}/\sqrt{Re}$ on $\delta/D$ and $Re$ becomes substantial, and when $\delta/D < 0.03$ and $Pr \to 2$, $Nu_{un}/\sqrt{Re}$ also changes appreciably depending on $\delta/D$.

The data obtained allow one to separate the unsteady-state stage of the process, when $Nu/\sqrt{Re}$ by more than 10% exceeds the steady-state value, from the quasi-steady state stage. The dimensionless time of the unsteady-state portion depends mainly on $Pr$ and can be estimated with the help of the expression

$$\tau_{\text{bound}} = 1.48/Pr^{0.43}.$$

In the case of water flow, at $Pr = 3$ and at the velocity of 1 $\text{ms}^{-1}$, past a cylinder of radius 1 m, $\tau^{*}_{bound} = 0.5$ s. During this time interval the heat transfer coefficient can exceed the steady-state value by tens of per cents, which can be substantial in short-time intense heat transfer processes.

The calculations qualitatively confirm the dependence of the heat transfer coefficient on time, thermophysical properties of the body material and the wall thickness, which was observed experimentally [17, 18] in unsteady-state conditions.

The time of a significant change (by more than 5%) in the complex $Nu/\sqrt{Re}$ is of the order of $10^{-5}$ s for air flow at $U_\infty = 300\,\text{ms}^{-1}$ past a sphere of radius $10^{-2}$ m and of order $10^{-2}$ s for water flow at $U_\infty = 1\,\text{ms}^{-1}$ past a sphere of the same radius. These values are much smaller than those found experimentally.

In practice, the cases of heat transfer between a flow and an obstacle are frequently encountered when a change in the flow parameters is close to a stepwise periodic one. An example is provided by the use of shutters which periodically cut off the flow from the heated barrier [20]. The mechanism of electric arc burning in a linear electric arc heater with short electrodes represents its blowing out and repeated striking at a large frequency [21], which leads to a periodic change in the temperature of the generated jet. Also, the cases of heat transfer are possible with a periodic change in the surface temperature.

The results of investigation of the barrier heat transfer characteristics on a model of the process representing a stepwise periodic change of the surface temperature between certain arbitrary values are given below. The period is divided into two time intervals during which the temperature remains constant. The temperature changes at the boundary between the intervals.

In order to solve this problem, the problem of the unsteady-state heat transfer at the frontal point of a sphere after a stepwise change in the flow temperature is first considered. Just as in [8, 10] the flow is assumed to be subsonic, laminar, steady; the properties of the liquid are assumed to be constant like the surface temperature. To solve the problem a system of equations (4)–(7) is used subject to the above assumptions.

The final state of the unsteady-state process investigated is the steady-state heat transfer regime.

It follows from the system of equations that the number $Nu$ in the present unsteady-state case of heat transfer is the function of numbers $Pr$, $Re$, and of dimensionless time $\tau$. Since the heat transfer in the body is not considered and the heat conduction equation is replaced, as is often the case [8, 10], by the surface condition, the parameters listed above lack the body characteristics of the reference ones, which generally influence the unsteady-state heat transfer rate [10, 22]. The $Pr$ number changed from 0.7 to 20, the time $\tau$ varied from zero up to the value at which the value of $Nu/\sqrt{Re}$ ceased to depend on $\tau$ with an error of 1–2%.

The developed heat transfer rates found (Fig. 5), to which the function $Nu/\sqrt{Re} = f(\tau)$ tends with the growth of time, represent the heat transfer characteristics in the steady-state conditions and are determined for $Pr = 0.7$–$20$ with the maximum approximation error of 3.5% at $Pr = 20$ by the expression

$$\frac{Nu_{st}}{\sqrt{Re}} = 1.33\,Pr^{0.362}. \tag{41}$$

This equation agrees well with formula (32) obtained for the problem of heat transfer in the stagnation point region in steady-state conditions [16].

The deviation of the values of $Nu_{st}/\sqrt{Re}$ found from formula (41) from those calculated by formula (32) within the range of $Pr$ numbers, for which the latter formula has been derived, does not exceed 2.3%.

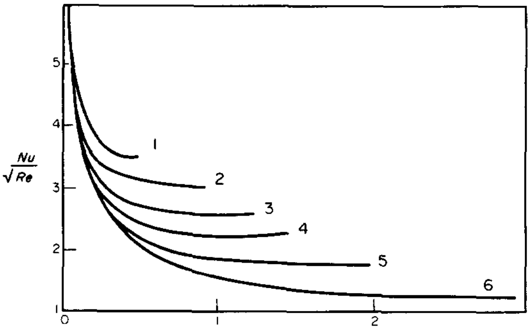


FIG. 5. The complex $Nu/\sqrt{Re}$ vs the dimensionless time $\tau$: 1, $Pr = 15$; 2, 10; 3, 6; 4, 4; 5, 2; 6, 0.7.

The results of calculation of the complex $Nu/\sqrt{Re}$, whose examples are given in Fig. 5, can be presented in the form of an approximate relation, whose approximation error does not exceed 1% at $Nu/\sqrt{Re} \to Nu_{st}/\sqrt{Re}$ and increases when $\tau \to 0$

$$\frac{Nu}{\sqrt{Re}} = \frac{Nu_{st}}{\sqrt{Re}}[1-\exp(0.2-1.7\,Pr^{0.35}\tau)]^{-1}. \quad (42)$$

The time of the unsteady-state portion of the process, when the instant of the steady-state heat transfer occurrence is governed by the condition $(Nu/\sqrt{Re})/(Nu_{st}/\sqrt{Re}) = 1.05$, is equal to

$$\tau_{bound} = 1.64\,Pr^{-0.35}. \quad (43)$$

For small values of $\tau$ at $Pr = 0.7$, the dependence of $Nu/\sqrt{Re}$ on $\tau$ can be represented by the expression

$$\frac{Nu}{\sqrt{Re}} = \frac{Nu_{st}}{\sqrt{Re}} + 0.5\,\tau^{-0.8}, \quad (44)$$

the error of approximation by which at $\tau = 0.05$–$0.5$ amounts respectively from 11 to 2%.

The obtained characteristics of the unsteady-state heat transfer can be used to estimate the mean characteristics of heat transfer in the case of a periodic stepwise variation of temperature.

Let the body surface (or flow) temperature at the initial instant of time change in a stepwise fashion to a new constant value and let this cause the unsteady state heat transfer process investigated above. After the time interval $kT$, equal to a portion of the period $T$, the body surface temperature changed to the initial value equal to the flow temperature and remained equal to this value during the second part of the period, during which there was no heat transfer between the body and the flow.

The mean (for the period) value of the complex $Nu/\sqrt{Re}$ depends substantially on the dimensionless period and can several times exceed the heat transfer rate in the steady-state conditions (Fig. 6). The effectiveness of enchancement, or the relative value of the complex $\overline{Nu/\sqrt{Re}}$, equal to

$$\bar{Z} = (\overline{Nu/\sqrt{Re}})/(Nu_{st}/\sqrt{Re}),$$

increases with a decrease of the $Pr$ number.

The values of the period $T$, at which the curves $\bar{Z} = f(T, Pr)$ intersect the straight line $\bar{Z} = 1$ in Fig. 6, can be called the critical ones, since at $T > T_{cr}$ the heat transfer rate, with a periodic change in temperature, becomes smaller than that of the respective steady-state process and, with an increase of the period, it tends to $k(Nu_{st}/\sqrt{Re})$. With an increasing frequency of the temperature change, the relative mean value of the complex $Nu/\sqrt{Re}$ increases significantly attaining the value of 3 at $T = 0.01$ and $Pr = 0.7$, i.e. with an increase in the frequency of temperature fluctuations, the heat transfer rate can substantially exceed this heat transfer characteristic in a corresponding steady-state process.

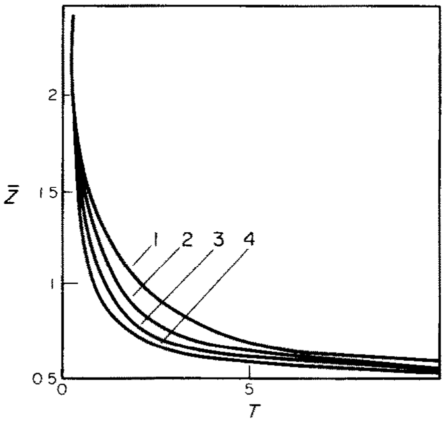


FIG. 6. The dependence of

$$\bar{Z} = \left[\int_0^{kT}(Nu/\sqrt{Re})\,d\tau\right]\Big/T(Nu_{st}/\sqrt{Re})$$

on the dimensionless period $T$: 1, $Pr = 0.7$; 2, 2; 3, 5; 4, 10; $k = 0.5$.

In the cases for which the model, adopted in this work, is applicable, the critical frequency of temperature fluctuations can be estimated for $Pr = 0.7 \div 10$ by the formula

$$f_{cr} = 1.68\,Pr^{-0.644}\,U_\infty R^{-1}. \quad (45)$$

In order to estimate $f_{cr}$ in the case of a barrier in the form of a plane disk of radius $R$, the coefficient 1.68 in formula (45) should be replaced by 0.71.

The dependence of $(\overline{Nu/\sqrt{Re}})/(Nu_{st}/\sqrt{Re})$ on the dimensionless period of temperature fluctuations and Pr number can be represented by the expression

$$\frac{\overline{Nu/\sqrt{Re}}}{Nu_{st}/\sqrt{Re}} = k + \frac{0.587}{Pr^{0.35}T} \times \ln\frac{1-\exp(-0.2-1.7\,Pr^{0.35}kT)}{0.181}. \quad (46)$$

In particular, in deriving formula (45) the coefficient $k$ was adopted to be 0.5.

It follows from the data presented that, e.g. at $Pr = 1$, $U_\infty = 100$ ms$^{-1}$, $R = 1$ m, $k = 0.5$, the critical frequency of fluctuations is 168 Hz. At the frequency of temperature fluctuations of 1 kHz, the heat transfer coefficient in these conditions is 1.5 times higher than that in steady-state conditions.

Thus, in the case of a viscous liquid flow at small flow velocities, the change in the heat transfer coefficient due to a single stepwise change of flow temperature outside boundary layer can be detected by the experimental method described in the literature. In an airflow, at high temperatures including, the change of $\alpha_{un}$ in oscillating pulsed processes can be appreciable for the same reason.

The calculations carried out have also shown that the thermophysical properties of the barrier material and its thickness markedly influence, alongside with the flow parameters and medium properties, the formation of the heat transfer characteristics in an unsteady-state process.

The condition of surface temperature being equal to zero during the unsteady-state process, which is usually adopted in theoretical investigations, is fulfilled at large values of $K'_\lambda$ at the initial stage of the process.

The time changes of the heat transfer coefficient observed experimentally cannot be explained by boundary layer relaxation after a stepwise change in the flow temperature.

It also follows from the results obtained that a periodic stepwise change of the flow temperature (or some approximation to it) can represent an effective mean of the heat transfer rate control.

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## RECHERCHE SUR LA TRANSFERT THERMIQUE VARIABLE DANS LA REGION DU POINT D'ARRET DANS DES ECOULEMENTS PLANS ET AXISYMETRIQUES

**Résumé**—L'article présente les résultats d'une étude expérimentale et théorique du transfert thermique sur la face avant d'une sphère et d'un cylindre en attaque frontale immergés dans un écoulement de gaz avec des conditions variables qui résultent d'un changement d'échelon de la température du fluide ou du corps.

## UNTERSUCHUNG DES INSTATIONÄREN WÄRMEÜBERGANGES IM BEREICH DES STAUPUNKTES EINER EBENEN ACHSENSYMMETRISCHEN STRÖMUNG

**Zusammenfassung**—Die Arbeit enthält die Ergebnisse einer experimentellen und theoretischen Untersuchung des Wärmeüberganges an der stromaufwärts gewandten Seite einer Kugel und eines quer angeordneten Zylinders. Diese sind einer instationären Gasströmung ausgesetzt, die durch eine sprunghafte Änderung der Gas- (oder Körper-) Temperatur zustande kommt.

## ИССЛЕДОВАНИЕ НЕСТАЦИОНАРНОГО ТЕПЛООБМЕНА В ОБЛАСТИ ТОЧКИ ТОРМОЖЕНИЯ ДЛЯ ПЛОСКОГО И ОСЕСИММЕТРИЧНОГО ТЕЧЕНИЙ

**Аннотация**—В работе приведены результаты экспериментального и теоретического исследования теплообмена в лобовой области сферы и поперечно обтекаемого цилиндра при обтекании тел потоком газа в нестационарных условиях, обусловленных ступенчатым изменением температуры потока (или тела).